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On the External Input Power into Coupled Structures

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Abstract

In the application of the statistical energy analysis (SEA) it is commonly assumed that the external input power is independent of the coupling between an externally driven structure (a master structure) and an attached passive structure (an adjunct structure). It is argued that although this assumption may be reasonable for weak couplings and, with some reservation, for strongly coupled similar structures, it may be incorrect for strongly coupled dissimilar structures. The definitions of similar and dissimilar coupled structures, in this context, are explained. The implication to SEA of this dissimilarity in strongly coupled structures is discussed in the light of developing noise control criteria for complexes that are composed of these coupled structures.

Introduction

It has been customary, when using the statistical energy analysis (SEA) to derive the response (stored energy) of a complex composed of coupled structures, to assume that the couplings among the structures do not influence the external input powers. In this sense, the external input powers, injected into the structures in isolation, are assumed to be invariants when couplings are instituted. This assumption initially simplified the analysis and subsequently its validity was not challenged, presumably a matter of "letting sleeping dogs lie". Indeed, as long as the couplings could be considered "weak" the assumption felt comfortably valid. Questions regarding the validity of the assumption arise only when "moderate" and, especially, "strong" couplings are brought in as a possibility. Chandiramani and Smith, Jr. attempted to account for changes in SEA as some of the couplings transit from weak to strong [1,2]. A major finding, in both efforts, is the convergence of the modal stored energies among those structures that are strongly coupled; as the couplings become stronger in these structures, there is a convergence toward equipartion of modal stored energies. In other words, as the coupling between two structures is increased, the coupled structures tend to merge and the stronger the coupling the more complete is the merger. In both papers, however, the influence, that the changes in the couplings may induce on the external input powers, is neglected apriori [1,2]. This neglect is examined in this paper; not rigorously, but, rather, heuristically. It is argued that for couplings among "similar" structures the external input powers are not dependent on the strength of the couplings. However, for "dissimilar" structures the external input powers are dependent on the strength of the couplings and there may then be a significant difference between those external input powers pertaining to weak and those pertaining to strong couplings.

II Rudimentary Statistical Energy Analysis (SEA)

The steady state equation of SEA for a complex composed of multiple coupled structures is

$$\underset{\approx}{\omega}\,\underset{\approx}{\eta}(\omega)\,\underset{\sim}{E}(\omega) = \underset{\sim}{\Pi}_e(\omega) \quad ; \quad \underset{\sim}{E}(\omega) = \{E_\alpha(\omega)\} \quad ; \quad \underset{\sim}{\Pi}_e(\omega) = \{\Pi_{e\alpha}(\omega)\} \quad ;$$

$$\underset{\approx}{\omega} = \left(\omega\delta_{\alpha\beta}\right) \quad ; \quad \underset{\approx}{\eta}(\omega) = \left(\left[\sum_{\gamma} \eta_{\gamma\alpha}(\omega)\right] \delta_{\alpha\beta} - \eta_{\alpha\beta}(\omega)\left(1 - \delta_{\alpha\beta}\right)\right) \quad , \tag{1a}$$

where (ω) is a center frequency of a frequency band of width $(\Delta\omega)$, $\eta_{\alpha\alpha}(\omega)$ is the loss factor associated with the (α)th structure, $\eta_{\gamma\alpha}(\omega)$ is the coupling loss factor associated with the attachment of the (α)th structure to the (γ)th, $E_\alpha(\omega)$ and $\Pi_{e\alpha}(\omega)$ are the stored energy and the external input power; in and into, the (α)th structure, respectively, and $\delta_{\alpha\beta}$ is the Kronecker delta. Significantly, Eq. (1a) is proper in the sense that $\underset{\approx}{\eta}(\omega)$ is a functional only of parameters that define the complex; it is independent of the elements of the stored energy vector $\underset{\sim}{E}(\omega)$ and of the external input power vector $\underset{\sim}{\Pi}_e(\omega)$. [Propriety is an essential ingredient to any successful analysis; for this reason many of the quantities and parameters involved are often tested for propriety.] The modal SEA quantities and parameters that are implicitly stated in Eq. (1a) are

$$E_\alpha(\omega) = \Delta\omega\, n_\alpha(\omega)\, \varepsilon_\alpha(\omega) \quad ; \quad \Pi_{e\alpha}(\omega) = \Delta\omega n_\alpha(\omega)\, \pi_{e\alpha}(\omega) \quad ;$$

$$[\eta_{\alpha\beta}(\omega)/\eta_{\beta\alpha}(\omega)] = \lambda^{\alpha}_{\beta}(\omega) = [n_\alpha(\omega)/n_\beta(\omega)] = [\lambda^{\beta}_{\alpha}(\omega)]^{-1} \quad , \tag{2}$$

where $n_\alpha(\omega)$ is the modal density, $\varepsilon_\alpha(\omega)$ is the averaged modal stored energy and $\pi_{e\alpha}(\omega)$ is the averaged modal external input power, all in reference to the (α)th structure [3]. The averaging is either over the frequency within the bandwidth $(\Delta\omega)$ or over an ensemble of complexes with differences that lie within that same bandwidth [3]. Since the "loss factor matrix" $\underset{\approx}{\eta}(\omega)$ is, by definition, nonsingular, Eq. (1a) may be inverted

$$\underset{\approx}{E}(\omega) = \underset{\approx}{\xi}(\omega)\,(\underset{\approx}{\omega})^{-1}\;\underset{\approx}{\Pi}_e(\omega) \quad ; \quad \underset{\approx}{\xi}(\omega) = \left(\xi_{\alpha\beta}(\omega)\right) = \left[\underset{\approx}{\eta}(\omega)\right]^{-1} \quad . \tag{1b}$$

The inverted loss factor matrix $\underset{\approx}{\xi}(\omega)$ may be dubbed the "gain factor matrix". Clearly, Eq. (1b), like Eq. (1a), is significantly proper. From Eq. (1a) one obtains

$$\sum_{\alpha} \omega\, \eta_{\alpha\alpha}(\omega)\, E_{\alpha}(\omega) = \Pi_e(\omega) \quad ; \quad \Pi_e(\omega) = \sum_{\alpha} \Pi_{e\alpha}(\omega) \quad , \tag{3a}$$

which simply expresses the equation of conservation of energy; it states that the dissipated power in the complex; $[\omega\eta_{\alpha\alpha}(\omega)E_{\alpha}(\omega)]$ in the (α)th structure, is equal to the external input power into the complex; $[\Pi_{e\alpha}(\omega)]$ into the (α)the structure. Similarly, from Eq. (1b) one obtains

$$E_{\alpha}(\omega) = \sum_{\gamma} E_{\alpha}^{\gamma}(\omega) \quad ; \quad E_{\alpha}^{\gamma}(\omega) = \xi_{\alpha\gamma}(\omega)\left[\Pi_{e\gamma}(\omega)/\omega\right] \quad , \tag{3b}$$

which simply accounts for the contributions, to the stored energy $E_{\alpha}(\omega)$ of the (α)th structure, by the external input powers into the various structures that compose the complex; e.g., $E_{\alpha}^{\gamma}(\omega)$ is contributed to $E_{\alpha}(\omega)$ by the external input power $[\Pi_{e\gamma}(\omega)]$ into the (γ)th structure.

An "effective loss factor" $\eta_e(\omega)$ may be defined in the form

$$\omega\eta_e(\omega)\, E(\omega) = \sum_{\alpha} \omega\eta_{\alpha\alpha}(\omega)\, E_{\alpha}(\omega) \quad ; \quad E(\omega) = \sum_{\alpha} E_{\alpha}(\omega) \quad . \tag{4a}$$

In this form $\eta_e(\omega)$ is a measure of the ability of the complex as a whole to dissipate the stored energy $E(\omega)$ in it [3-5]. Casting Eq. (4a) in the algebraically manipulated form

$$\eta_e(\omega) = \left[\sum_{\alpha} \eta_{\alpha\alpha}(\omega)\, E_{\alpha}(\omega) \,/\, \sum_{\beta} E_{\beta}(\omega)\right] \quad , \tag{4b}$$

interprets the effective loss factor $\eta_e(\omega)$ to be the "average loss factor"; averaged over the stored energies of the structures comprising the complex. Similarly, an "effective gain factor" $\xi_{e\alpha}(\omega)$ is

defined in the form

$$\xi_{e\alpha}(\omega)\left[\Pi_e(\omega)/\omega\right] = \sum_{\gamma} \xi_{\alpha\gamma}(\omega)\left[\Pi_{e\gamma}(\omega)/\omega\right] \quad . \tag{5a}$$

In this form $\xi_{e\alpha}(\omega)$ is a measure of the transfer of energy from the external input powers to the (α)th structure. Casting Eq. (5a) in the form

$$\xi_{e\alpha}(\omega) = \Pi_{e\gamma}(\omega)\left[\sum_{\gamma} \xi_{\alpha\gamma}(\omega)\,\Pi_{e\gamma}(\omega)/\sum_{\beta}\Pi_{e\beta}(\omega)\right] \quad , \tag{5b}$$

interprets the effective gain factor $\xi_{e\alpha}(\omega)$ of the (α)th structure to be the "average gain factor" of that structure; averaged over the external input powers of the structures comprising the complex. From Eqs. (3a) and (4) one obtains

$$\omega\eta_e(\omega)\,E(\omega) = \Pi_e(\omega) \quad ; \quad \eta_e(\omega) = \left[\Pi_e(\omega)/\omega E(\omega)\right] \quad , \tag{6a}$$

and from Eqs. (3b) and (5) one obtains

$$\xi_{e\alpha}(\omega)\left[\Pi_e(\omega)/\omega\right] = E_\alpha(\omega) \quad ; \quad \xi_{e\alpha}(\omega) = \left[\omega\; E_\alpha(\omega)/\Pi_e(\omega)\right] \quad . \tag{6b}$$

From Eqs. (6a) and (6b), the identity

$$\eta_e(\omega) = \left[\xi_e(\omega)\right]^{-1} \quad ; \quad \xi_e(\omega) = \sum_{\alpha} \xi_{e\alpha}(\omega) \quad , \tag{7}$$

emerges.

As usual, for good and bad reasons, a complex consisting of merely two structures is chosen to demonstrate the notions and concepts that lie within SEA and to illustrate results obtained by rendering SEA to such a complex. A SEA model of a two-structures complex is depicted in

Fig. 1; one structure is designated the (s)th structure (the master structure) and the other the (b)th structure (the fuzz in a structural fuzzy) [4-6]. A question is then posed: May an externally driven structure (the (s)th structure) be adjoined by another (the (b)th structure) in order to improve the noise control integrity of the union; e.g., by increasing the effective loss factor $\eta_e(\omega)$ of the combined structures beyond that of the initial structure? For the two-structures complex Eq. (4b), after straightforward algebraic manipulations, yields

$$\eta_{es}(\omega) = \left[\eta_e(\omega)/\eta_{ss}(\omega)\right] = \left[1 + \eta_s^b(\omega)\,\zeta_s^b(\omega)\right]\left[1 + \zeta_s^b(\omega)\right]^{-1} \quad ;$$

$$\eta_s^b(\omega) = \left[\eta_{bb}(\omega)/\eta_{ss}(\omega)\right] \quad ; \quad \zeta_s^b(\omega) = \left[E_b(\omega)/E_s(\omega)\right] \quad . \tag{4c}$$

Using Eq. (4c), Fig. 2 is presented. In this figure values of $\eta_{es}(\omega)$ are depicted as a function of $\eta_s^b(\omega)$ and $\zeta_s^b(\omega)$. It is revealed that for a practical range of $\{\eta_s^b(\omega), \zeta_s^b(\omega)\}$ there exist a region in which $\eta_{es}(\omega)$ exceeds unity, notwithstanding that in another region, $\eta_{es}(\omega)$ is less than unity. The region in Fig. 2 for which $\eta_{es}(\omega) \gtrsim \sqrt{10}$, establishes a set of criteria that needs to be satisfied by parameters that define the two-structures complex to affirmatively answer the question posed. However, as argued in Reference 4, that $\eta_{es}(\omega)$ exceeds unity may be a necessary, but is not a sufficient condition for achieving an effective noise control. To achieve and effective noise control a more comprehensive analysis, than that involved in the determination of $\eta_{es}(\omega)$, is required

II Stored Energy Ratios in Noise Control Criteria

To examine more comprehensively the noise control criteria it is convenient to model the complex in terms of unattached structures. Whereas Eq. (1) is the insitu description of the complex, the equation of SEA for a complex in which the structures are artificially, but appropriately, isolated from each other is

$$\underset{\approx}{\omega}\, \underset{\approx}{\eta}^0(\omega)\, \underset{\sim}{E}^0(\omega) = \overset{0}{\underset{\sim}{\Pi}}_e(\omega) \quad ; \quad \underset{\sim}{E}^0(\omega) = \{E^0_\alpha(\omega)\} \quad ; \quad \overset{0}{\underset{\sim}{\Pi}}_e(\omega) = \{\overset{0}{\Pi}_{e\alpha}(\omega)\} \quad ;$$

$$\underset{\approx}{\omega} = \left(\omega\delta_{\alpha\beta}\right) \quad ; \quad \underset{\approx}{\eta}^0(\omega) = \left(\overset{0}{\eta}_{\alpha\alpha}(\omega)\,\delta_{\alpha\beta}\right) \quad , \tag{8a}$$

or invertedly

$$\underset{\sim}{E}^0(\omega) = \overset{0}{\underset{\approx}{\xi}}(\omega)\, \underset{\approx}{\omega}^{-1}\, \overset{0}{\underset{\sim}{\Pi}}_e(\omega) \quad ; \quad \overset{0}{\underset{\approx}{\xi}}(\omega) = \left(\overset{0}{\xi}_{\alpha\alpha}(\omega)\,\delta_{\alpha\beta}\right) \quad ; \quad \overset{0}{\underset{\approx}{\xi}}(\omega) = [\underset{\approx}{\eta}^0(\omega)]^{-1} \quad . \tag{8b}$$

A typical equation of motion for a structure may be selected from Eq. (8). It reads

$$\omega\overset{0}{\eta}_{\alpha\alpha}(\omega)\, E^0_\alpha(\omega) = \overset{0}{\Pi}_{e\alpha}(\omega) \quad ; \quad E^0_{\alpha\alpha}(\omega) = \overset{0}{\xi}_{\alpha\alpha}(\omega)\, [\overset{0}{\Pi}_{e\alpha}(\omega)/\omega] \quad . \tag{9}$$

This equation states that the stored energy $E^0_\alpha(\omega)$ is attained by the isolated (α)th structure in response to an external input power $\overset{0}{\Pi}_{e\alpha}(\omega)$ that is directly injected into it; this power is dissipated, in this structure in isolation, by the loss factor $\overset{0}{\eta}_{\alpha\alpha}(\omega)$. From Eqs. (1b) and (8) one may define three useful forms of stored energy ratios: $\mathcal{E}_\alpha(\omega)$, $\mathcal{E}^\gamma_{0\alpha}(\omega)$ and $\mathcal{E}^\gamma_\alpha(\omega)$. The first ratio is defined

$$\mathcal{E}_\alpha(\omega) = [E_\alpha(\omega)/E^0_\alpha(\omega)] = \xi^\alpha_0(\omega)\, P^\alpha_0(\omega) \quad ; \quad \xi^\alpha_0(\omega) = [\xi_{\alpha\alpha}(\omega)\, \overset{0}{\eta}_{\alpha\alpha}(\omega)] \quad ;$$
$$P^\alpha_0(\omega) = [\Pi_{e\alpha}(\omega)/\overset{0}{\Pi}_{e\alpha}(\omega)] \quad , \tag{10a}$$

and may be used, for example, to assess the influence, on the stored energy of the (α)th structure,

due to the incorporation of this structure to be an insitu member of the complex. Clearly $\mathcal{E}_{\alpha}(\omega)$ is proportional to the ratio $P_0^{\alpha}(\omega)$ of the insitu external input power into the (α)th structure to that in isolation; i.e., before it is incorporated in the complex. The second stored energy ratio is defined

$$\mathcal{E}_{0\alpha}^{\gamma}(\omega) = [E_{\alpha}^{\gamma}(\omega)/E_{\alpha}^{0}(\omega)] = \xi_0^{\alpha\gamma}(\omega)\, P_{0\alpha}^{\gamma}(\omega) \quad ; \quad \xi_0^{\alpha\gamma}(\omega) = [\xi_{\alpha\gamma}(\omega)\, \overset{0}{\eta}_{\alpha\alpha}(\omega)] \quad ;$$

$$P_{0\alpha}^{\gamma}(\omega) = [\Pi_{e\gamma}(\omega)/\overset{0}{\Pi}_{e\alpha}(\omega)] \quad , \tag{10b}$$

and may be used, for example, to assess the influence, on the stored energy of the (α)th structure, due to the incorporation of this structure to be an insitu member of the complex when the external input power into that complex is applied to another structure; e.g., to the (γ)th structure. Clearly $\mathcal{E}_{0\alpha}^{\gamma}(\omega)$ is proportional to the ratio $P_{0\alpha}^{\gamma}(\omega)$ of the external input power into the (γ)th structure insitu and the external input power into the (α)th structure in isolation. The third ratio is defined

$$\mathcal{E}_{\alpha}^{\gamma}(\omega) = [\mathcal{E}_{0\alpha}^{\gamma}(\omega)/\mathcal{E}_{\alpha}(\omega)] = [E_{\alpha}^{\gamma}(\omega)/E_{\alpha}(\omega)] = \xi_{\alpha}^{\alpha\gamma}(\omega)\, P_{\alpha}^{\gamma}(\omega) \quad ;$$

$$\xi_{\alpha}^{\alpha\gamma}(\omega) = [\xi_{\alpha\gamma}(\omega)/\xi_{\alpha\alpha}(\omega)] \quad ; \quad P_{\alpha}^{\gamma}(\omega) = [\Pi_{e\gamma}(\omega)/\Pi_{e\alpha}(\omega)] \quad , \tag{10c}$$

and may be used, for example, to assess the influence, on the stored energy of the (α)th structure, of injecting the external input power into the (γ)th structure versus injecting it into the (α)th structure itself. The ratio between these two external input powers is designated $P_{\alpha}^{\gamma}(\omega)$ and the stored energy ratio $\mathcal{E}_{\alpha}^{\gamma}(\omega)$ is proportional to this external input power ratio under the conditions just specified.

If noise control of the (α)th structure is the criterion of import, then the desire is to minimize, in each case, the one relevant ratio of the three; either $\mathcal{E}_{\alpha}(\omega)$, $\mathcal{E}_{0\alpha}^{\gamma}(\omega)$ or $\mathcal{E}_{\alpha}^{\gamma}(\omega)$. Minimization of this kind renders the selected ratio small compared with unity; the smaller the more commendable is the noise control achievement. The first factor in each of these three stored energy ratios; namely, $\xi_0^{\alpha}(\omega)$, $\xi_0^{\alpha\gamma}(\omega)$ and $\xi_{\alpha}^{\alpha\gamma}(\omega)$, respectively, are proper quantities. The propriety is in

the sense that these quantities are functional only of the properties of the structures and the couplings among them; they are independent of the stored energies in, and the external input powers into the individual structures that comprise the complex. These properties, in SEA, are defined in terms of the elements of the loss factor matrix $\underset{\approx}{\eta}(\omega)$ and/or of the gain factor matrix $\underset{\approx}{\xi}(\omega)$. For a prescribed model of the complex, these elements are assumed known and changes in these elements, to achieve a desired noise control condition, are also assumed known. A question arises with respect to the second factor in each of the three stored energy ratios: Are these second factors; namely, $P_0^{\alpha}(\omega)$, $P_{0\alpha}^{\gamma}(\omega)$ and $P_{\alpha}^{\gamma}(\omega)$, respectively, which consist of various ratios of external input powers into specific structures, directly or indirectly dependent on the properties of the structures and especially of the couplings among them? If the external input powers are assumed to be independent of the couplings

$$P_0^{\alpha}(\omega) \Rightarrow P_0^{0\alpha}(\omega) = 1 \quad , \tag{11a}$$

$$P_{0\alpha}^{\gamma}(\omega) \Rightarrow P_{0\alpha}^{0\gamma}(\omega) = [\lambda_{\alpha}^{\gamma}(\omega)\,(M_{\alpha}/M_{\gamma})] \quad , \tag{11b}$$

$$P_{\alpha}^{\gamma}(\omega) \Rightarrow P_{\alpha}^{0\gamma}(\omega) = P_{0\alpha}^{0\gamma}(\omega) \quad . \tag{11c}$$

On the other hand, if the external input powers are dependent on the couplings, changes in these couplings, which are designed to achieve desired noise control condition with respect to the first factors, may influence, adversely or beneficially, the corresponding second factors. The central theme of this paper is the investigation of the influence of the couplings on these second factors.

III External Input Power at SEA

It is usual to specify the external drive, to which a structure may be subjected, by an external force-source or an external velocity-source; in the first the external force is specified, in the second the externally imposed velocity is specified. When the external force-source is employed the external input power into a structure is

$$\Pi_e(\omega) = \left\langle |F_e(\omega)|^2 \right\rangle \left\langle G(\omega) \right\rangle \quad ; \quad S_e(\omega)\Delta\omega = 2\pi \left\langle |F_e(\omega)|^2 \right\rangle \quad ;$$

$$\left\langle G(\omega) \right\rangle = \left[(\pi/2)\, n(\omega)/M \right] \quad , \tag{12a}$$

and when the external velocity-source is employed the external input power into a structure is

$$\Pi_e(\omega) = \left\langle |V_e(\omega)|^2 \right\rangle \left\langle [G(\omega) - iB(\omega)]^{-1} \right\rangle \quad ; \tag{12b}$$

where $G(\omega)$ is the conductance and $B(\omega)$ is the susceptance, with both these quantities being real, $S_e(\omega)$ is the quadratic spectral distribution of the force-source, $n(\omega)$ is the modal density and M is the mass of the structure [3]. In order to avoid difficulties, without loss in insight, Eq. (12a) is used in this paper and the use of Eq. (12b) is deferred to another. Figure 3a depicts the single structure on which attention is now focused. The SEA equation of motion for this structure is given by

$$\omega\eta(\omega)E(\omega) = \Pi_e(\omega) \quad , \tag{13}$$

where $\eta(\omega)$ is the loss factor and $E(\omega)$ is the stored energy that is generated by the external input power $\Pi_e(\omega)$. The external input power $\Pi_e(\omega)$ into this structure is stated, as agreed, in Eq. (12a). It may be instructive to subdivide this structure into two substructures; one designated (1) and the other (2), as prescribed in Fig. 3b. The SEA equation of motion for the two-structures complex is

given by

$$\omega\eta_{11}(\omega)E_1(\omega) + \omega\eta_{22}(\omega)\, E_2(\omega) = \Pi_{e1}(\omega) \quad , \tag{14a}$$

$$\omega[\eta_{22}(\omega) + \eta_{12}(\omega)]\, E_2(\omega) - \omega\eta_{21}(\omega)\, E_1(\omega) \quad , \tag{14b}$$

$$\eta_{e1}(\omega) = [\eta_e(\omega)/\eta_{11}(\omega)] = [1 + \eta_1^2(\omega)\, \zeta_1^2(\omega)]\ [1 + \zeta_1^2(\omega)]^{-1} \quad ;$$
$$\eta_1^2(\omega) = [\eta_{22}(\omega)/\eta_{11}(\omega)] \quad ; \quad \zeta_1^2(\omega) = [E_2(\omega)/E_1(\omega)] \quad . \tag{14c}$$

[cf. Eqs. (1a) and (4).] Invoking Eq. (2), Eq. (14c) may be cast in terms of averaged modal quantities and parameters

$$\eta_{e1}(\omega) = [\eta_e(\omega)/\eta_{11}(\omega)] = \left\{1 + \eta_1^2(\omega)\, [\lambda_1^2(\omega)\, \sigma_1^2(\omega)]\right\} \left\{1 + [\lambda_1^2(\omega)\, \sigma_1^2(\omega)]\right\}^{-1} \quad ;$$

$$\zeta_1^2(\omega) = [\lambda_1^2(\omega)\, \sigma_1^2(\omega)] \quad ; \quad \sigma_1^2(\omega) = [\varepsilon_2(\omega)/\varepsilon_1(\omega)] = [\nu_{12}^2(\omega) + 1]^{-1} \quad ;$$

$$\lambda_1^2(\omega) = [n_2(\omega)/n_1(\omega)] = [\eta_{21}(\omega)/\eta_{12}(\omega)] \quad ; \quad \nu_{12}^2(\omega) = [\eta_{22}(\omega)/\eta_{12}(\omega)] \quad . \tag{15}$$

Thus, the "coupling quotient" $\nu_{12}^2(\omega)$, the modal density ratio $\lambda_1^2(\omega)$, the modal stored energy ratio $\sigma_1^2(\omega)$ and the effective loss factor ratio $\eta_{e1}(\omega)$ are all proper parameters of the two-structures complex. Clearly, if $\nu_{12}^2(\omega) >> 1$, the coupling between the (2)th and (1)th structures, in this complex, is weak and $\sigma_1^2(\omega) << 1$; if $10 \gtrsim \nu_{12}^2(\omega) \lesssim 1$, the coupling is moderate and $10^{-1} \lesssim \sigma_1^2(\omega) \lesssim (1/2)$; and if $\nu_{12}^2(\omega) \lesssim 1$, the coupling is strong and $\sigma_1^2(\omega) \rightarrow 1$. The classification is in accord with previous treatises on this subject [1,2,7,8]. This is not, however, the whole story; the material discussed in Section II is called upon to contribute to the story too. In this vein the (1)th structure is appropriately isolated and its SEA equation of motion is then stated in the form

$$\omega \eta_{11}^{0}(\omega)\, E_1^0(\omega) = \overset{0}{\Pi}_{e1}(\omega) \quad , \tag{16a}$$

and if the external force-source remains unaltered by the isolation, the external input power injected into the (1)th structure in isolation is

$$\overset{0}{\Pi}_{e1}(\omega) = \langle |F_e(\omega)|^2 \rangle \langle G_1^0(\omega) \rangle \quad ; \quad \langle G_1^0(\omega) \rangle = (\pi/2)\, [n_1(\omega)/M_1] \quad . \tag{16b}$$

[cf. Eqs. (8) and (12a), and Fig. 3b.] From Eqs. (10a) and (14) through (16) one derives

$$\mathcal{E}_1(\omega) = [E_1(\omega)/E_1^0(\omega)] = \xi_0^1(\omega)\, P_0^1(\omega) \quad , \tag{17}$$

$$\xi_0^1(\omega) = [\xi_{11}(\omega)\, \eta_{11}^{0}(\omega)] \quad ; \quad \xi_{11}(\omega) = \left\{ \eta_e(\omega) \left(1 + [\lambda_1^2(\omega)\, \sigma_1^2(\omega)] \right) \right\}^{-1} \quad , \tag{18a}$$

$$P_0^1(\omega) = [\Pi_{e1}(\omega)/\overset{0}{\Pi}_{e1}(\omega)] \quad . \tag{18b}$$

To set the stage, it is contrived that the division of the structure into a two-structures complex is constructed at a "controlled boundary"; i.e., at a thought boundary that involves no change in the physical properties of the original structure. Under this construction SEA would demand that quantities and parameters in Eqs. (12) and (13) match those in Eqs. (17) and (18) in the form

$$\sigma_1^2(\omega) \to 1 \quad ; \quad \eta(\omega) \to \eta_e(\omega) \quad ; \quad \Pi_e(\omega) \to \Pi_{e1}(\omega) \quad . \tag{19}$$

The first of these matchings signifies that the couplings at the controlled boundary is strong, as a merged structure would; after all, the complex is, in fact, a single structure! From Eqs. (17) through (19) one finds

$$\mathcal{E}_1(\omega) = [\eta^0_{11}(\omega)/\eta(\omega)]\,[M_1/M] \quad ; \quad M = M_1 + M_2 \quad , \tag{20}$$

$$\xi^1_0(\omega) = [\eta^0_{11}(\omega)/\,\eta(\omega)]\,[n_1(\omega)/n(\omega)] \quad ; \quad n(\omega) = n_1(\omega) + n_2(\omega) \quad , \tag{21a}$$

$$P^1_0(\omega) = [n(\omega)/M]\ \ [M_1/n_1(\omega)] \ = \ [1 + \lambda^2_1(\omega)]\ \ [1 + (M_2/M_1)]^{-1} \quad . \tag{21b}$$

Two major points emerged: The first is the simplicity of Eq. (20) and the second is the particular form of $P^1_0(\omega)$, as stated in Eq. (21b). This particular form of $P^1_0(\omega)$ may be summarized

$$P^1_0(\omega) \begin{cases} > 1 \quad , \quad \lambda^2_1(\omega) > (M_2/M_1) & , \ (22a) \\ = 1 \quad , \quad \lambda^2_1(\omega) = (M_2/M_1) & , \ (22b) \\ < 1 \quad , \quad \lambda^2_1(\omega) < (M_2/M_1) & , \ (22c) \end{cases}$$

and one is reminded that $\lambda^2_1(\omega) = [n_2(\omega)/n_1(\omega)]$. Equation (22b) defines the two structures; (1)th structure and (2)th structure, to be similar and Eqs. (22a) and (22c) define them to be dissimilar; Eq. (22a) defines a "light" and Eq. (22c) defines a "heavy" adjoined (2)th structure. In Eq. (22) the notion that the external input power into a structure is independent of the coupling when another is adjoined to it, is challenged, notwithstanding that if the structures are similar, as just defined, the challenge is muted.

The contrived division of the structure, just discussed, brings out another topic of significance. The division clearly results in two structures, each of which has a resonance frequency distribution that occupies a higher region of the ω-domain than the original. Indeed, in the lower frequency region, where "global-modes" lie, the division of the single structure cannot be strictly entertained. As suggested by the global-modes designation, in that lower frequency region in which these global-modes reside, the (1)th and (2)th structures must be assumed merged apriori. This merger, in turn, validates apriori the matching, stated in Eq. (19), for the global-modes.

Returning to the central topic of discussion, the boundary between the two structures is assumed physical rather than controlled, and, therefore, Eq. (19) could no longer be validated apriori. One expects that if the coupling of the (2)th structure to the (1)th structure, at this boundary, is weak, the external input power ratio $P_0^1(\omega)$ converges onto unity. Heuristically, the bridge between weak and strong coupling, and vice versa, can be expressed, for the two-structures complex, in the form

$$P_0^1(\omega) = \left\{1 + \lambda_1^2(\omega)\,[\sigma_1^2(\omega)]^q\right\}\left\{1 + (M_2/M_1)\,[\sigma_1^2(\omega)]^q\right\}^{-1} \quad ;$$

$$\sigma_1^2(\omega) = [1 + v_{12}^2(\omega)]^{-1} < 1 \quad , \tag{23a}$$

where the index q is yet to be determined; a likely candidate, however, is q = 1. In Eq. (23a) a weak coupling is characterized by a small value, compared with unity, for $\sigma_1^2(\omega)$ and, in accord with Eq. (19), a strong coupling is characterized by a value of $\sigma_1^2(\omega)$ that converges on unity. In any case, for the two-structures complex, as Eq. (15) attests, the quantity $\sigma_1^2(\omega)$ is a proper parameter. In this sense Eq. (23a) also defines a proper quantity; $P_0^1(\omega)$ is proper. Using Eq. (23a), Fig. 4 is presented. In this figure values of $P_0^1(\omega)$ are depicted as a function of (M_2/M_1) and $\lambda_1^2(\omega)$ for two fixed values of $\sigma_1^2(\omega)$; in Fig. 4a, $\sigma_1^2(\omega) = 1$, which is commensurate with strong coupling and in Fig. 4b, $\sigma_1^2(\omega) = 0.2$, which is commensurate with a fairly weak coupling. Analogously, the external input power ratio $P_{01}^2(\omega)$, as stated in Eqs. (10b) and (12a), can be expressed, for the two-structures complex, in the form

$$P_{01}^2(\omega) = P_{01}^{02}(\omega)\left\{1 + \lambda_2^1(\omega)\,[\sigma_2^1(\omega)]^q\right\}\left\{1 + (M_1/M_2)\,[\sigma_2^1(\omega)]^q\right\}^{-1} \;;$$

$$P_{01}^{02}(\omega) = [\lambda_1^2(\omega)\,(M_1/M_2)] \;\; ; \;\; \sigma_2^1(\omega) = [1 + v_{21}^1(\omega)]^{-1} < 1 \quad ;$$

$$v_{21}^1(\omega) = [\eta_{11}(\omega)/\eta_{21}(\omega)] \quad , \tag{23b}$$

and again, $P_{01}^2(\omega)$, in Eq. (23b), as Eq. (15) attests, is proper. [cf. Eq. (11b).] The criterion for

weak or strong coupling in Eq. (23b) is characterized by whether $\sigma_2^1(\omega)$ is small compared with unity or approaches unity, respectively. Finally, from Eqs. (23a) and (23b) and Eqs. (10c) and (12a) one obtains

$$P_1^2(\omega) = \left\{ [P_{01}^2(\omega)]\ (\sigma_2^1(\omega) < 1) / [P_0(\omega)]\ (\sigma_1^2(\omega) < 1) \right\} \quad , \tag{23c}$$

and, clearly, $P_1^2(\omega)$ is proper. [The product or the ratio of two proper quantities is proper!] From Eq. (23) it emerges, again, that if the two structures are similar

$$\lambda_1^2(\omega) = (M_2/M_1) \quad , \tag{24}$$

the external input power ratios, just stated, are all substantially equal to unity, independently of the coupling [1,2,4]. It also emerges, again, that if the two structures are dissimilar and the coupling is strong in the sense that the coupling quotient $v_{12}^2(\omega)$ is small compared with unity so that $\sigma_1^2(\omega) \rightarrow 1$, the external input power ratio $P_0^1(\omega)$ differs from unity. Notwithstanding that for weak coupling in the sense that the coupling quotient $v_{12}^2(\omega)$ is large compared with unity so that $\sigma_2^1(\omega) << 1$, $P_0^1(\omega)$ approaches unity even if the structures are not similar; i.e., when Eq. (23) is violated. Analogeous assessment can be conducted with respect to $P_{01}^2(\omega)$, stated in Eq. (23b), and $P_1^2(\omega)$, stated in Eq. (23c).

The generalization of Eq. (23) can be readily made. Following the definitions of $P_0^\alpha(\omega)$, $P_{0\alpha}^\gamma(\omega)$ and $P_\alpha^\gamma(\omega)$, stated in Eq. (10), utilizing Eq. (12a) and the procedure that led to Eq. (23), one may readily extrapolate and state

$$P_0^\alpha(\omega) = \left\{ \sum_\beta \lambda_\alpha^\beta(\omega)\, [\sigma_\alpha^\beta(\omega)]^q / \sum_\beta (M_\beta/M_\alpha)\, [\sigma_\alpha^\beta(\omega)]^q \right\} \quad ;$$

$$\sigma_\alpha^\beta(\omega) = [\xi_{\beta\alpha}(\omega)/\lambda_\alpha^\beta(\omega)\, \xi_{\alpha\alpha}(\omega)] < 1 \quad , \tag{25a}$$

$$P^{\gamma}_{0\alpha}(\omega) = P^{0\gamma}_{0\alpha}(\omega) \left\{ \sum_{\beta} \lambda^{\beta}_{\gamma}(\omega) [\sigma^{\beta}_{\gamma}(\omega)]^{q} / \sum_{\beta} (M_{\beta}/M_{\gamma}) [\sigma^{\beta}_{\gamma}(\omega)]^{q} \right\} \quad ;$$

$$P^{0\gamma}_{0\alpha}(\omega) = [\lambda^{\gamma}_{\alpha}(\omega) (M_{\alpha}/M_{\gamma})] \quad ; \quad \sigma^{\beta}_{\gamma}(\omega) = [\xi_{\beta\gamma}(\omega)/\lambda^{\beta}_{\gamma}(\omega)\, \xi_{\gamma\gamma}(\omega)] < 1 \quad , \tag{25b}$$

$$P^{\gamma}_{\alpha}(\omega) = [P^{\gamma}_{0\alpha}(\omega)](\sigma^{\beta}_{\gamma}(\omega) < 1) / [P^{\alpha}_{0}(\omega)](\sigma^{\beta}_{\alpha}(\omega) < 1) \quad , \tag{25c}$$

where use is made of Eqs. (1b) and (2) and $P^{0\gamma}_{0\alpha}(\omega)$ is defined in Eq. (11b). Since the elements in the gain factor matrix $\underset{\approx}{\xi}(\omega)$ are proper and so are the modal density ratios, the external input power ratios, stated in Eq. (25), are also proper. Thus, in this generalization to a multi (more than two)-structures complex the quantities $P^{\alpha}_{0}(\omega)$, $P^{\gamma}_{0\alpha}(\omega)$ and $P^{\gamma}_{\alpha}(\omega)$ are proper, notwithstanding that once the first two are, the propriety of the third follows. Moreover, it remains invariant that if the structures are similar

$$\lambda^{\beta}_{\alpha}(\omega) = (M_{\beta}/M_{\alpha}) \quad , \tag{26}$$

the external input power ratios, stated in Eq. (25), are all equal to unity. When the structures are dissimilar, deviations from unity of these ratios may occur; such deviations from unity need to be estimated when noise control criteria are being developed for multi-structures complexes [4,8]. Of particular import in these estimates is the influence of the couplings, among the various structures, on these ratios. The manner of estimating these deviations, as they relate to the couplings among the structures, is presented in this paper. Equation (25) in conjunction with Eq. (10) may be efficaciously employed to establish noise control criteria in situations in which dissimilar structures are coupled memebers in the same complex.

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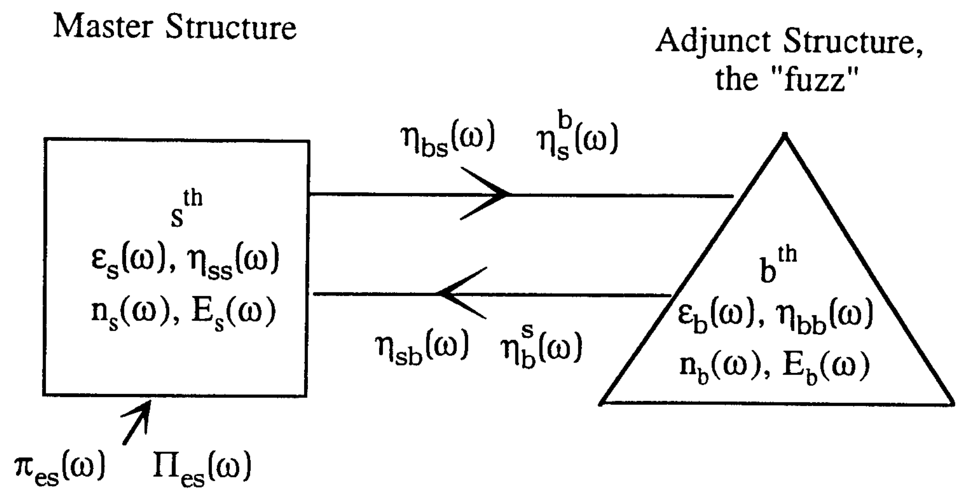


Figure 1, A SEA model of two coupled structures.

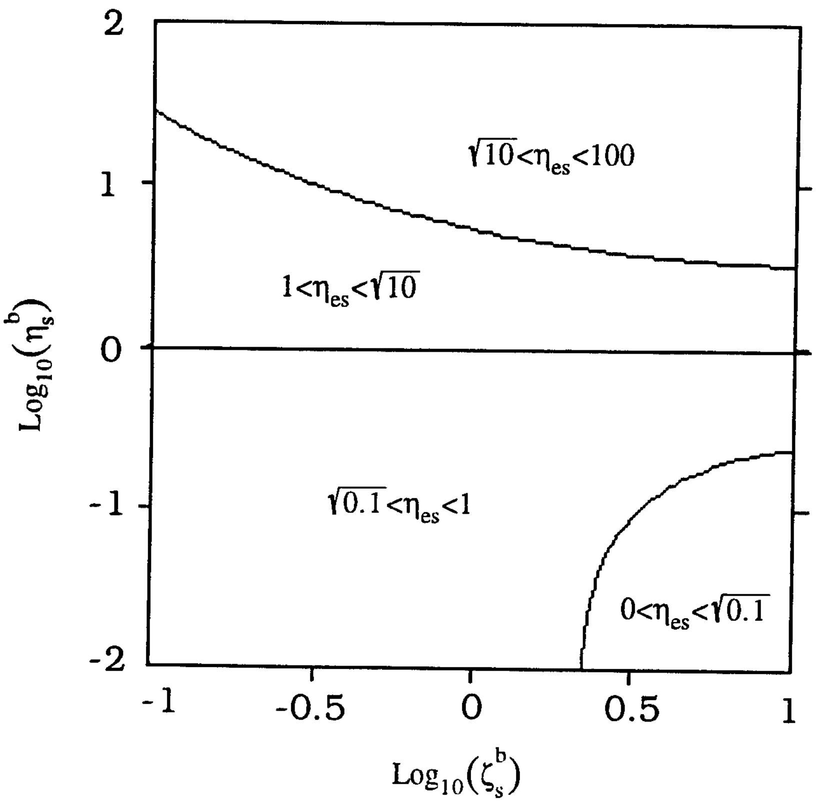
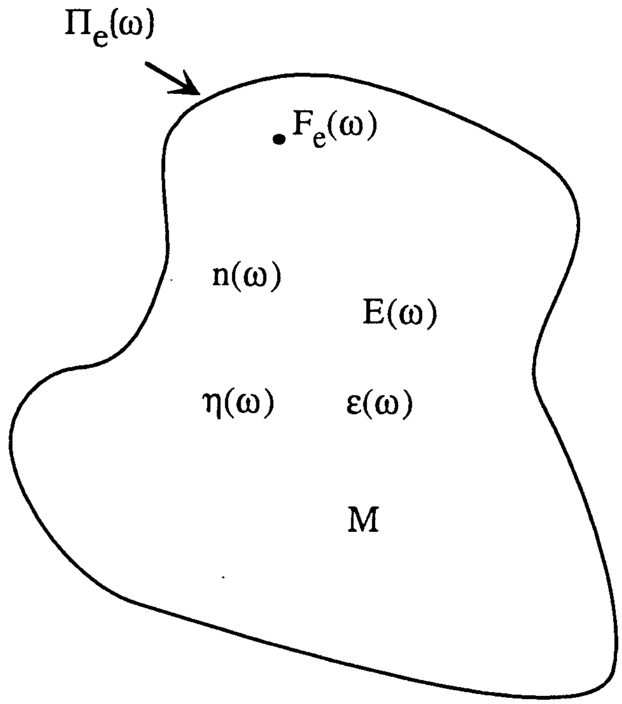
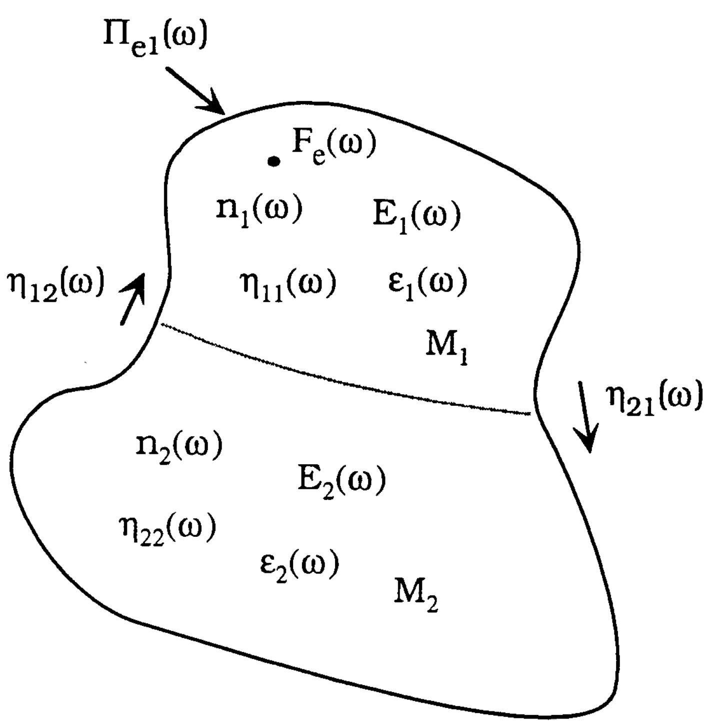


Fig. 2 The effective loss factor ratio $\eta_{es}(\omega)$ as a function of the stored energy ratio $\zeta_s^b(\omega)$ and the loss factor ratio $\eta_s^b(\omega)$.



a) An externally driven SEA model of a simple structure.



b) A two-structure complex obtained by appropriate division of the structure depicted in Fig. 3a.

Fig. 3

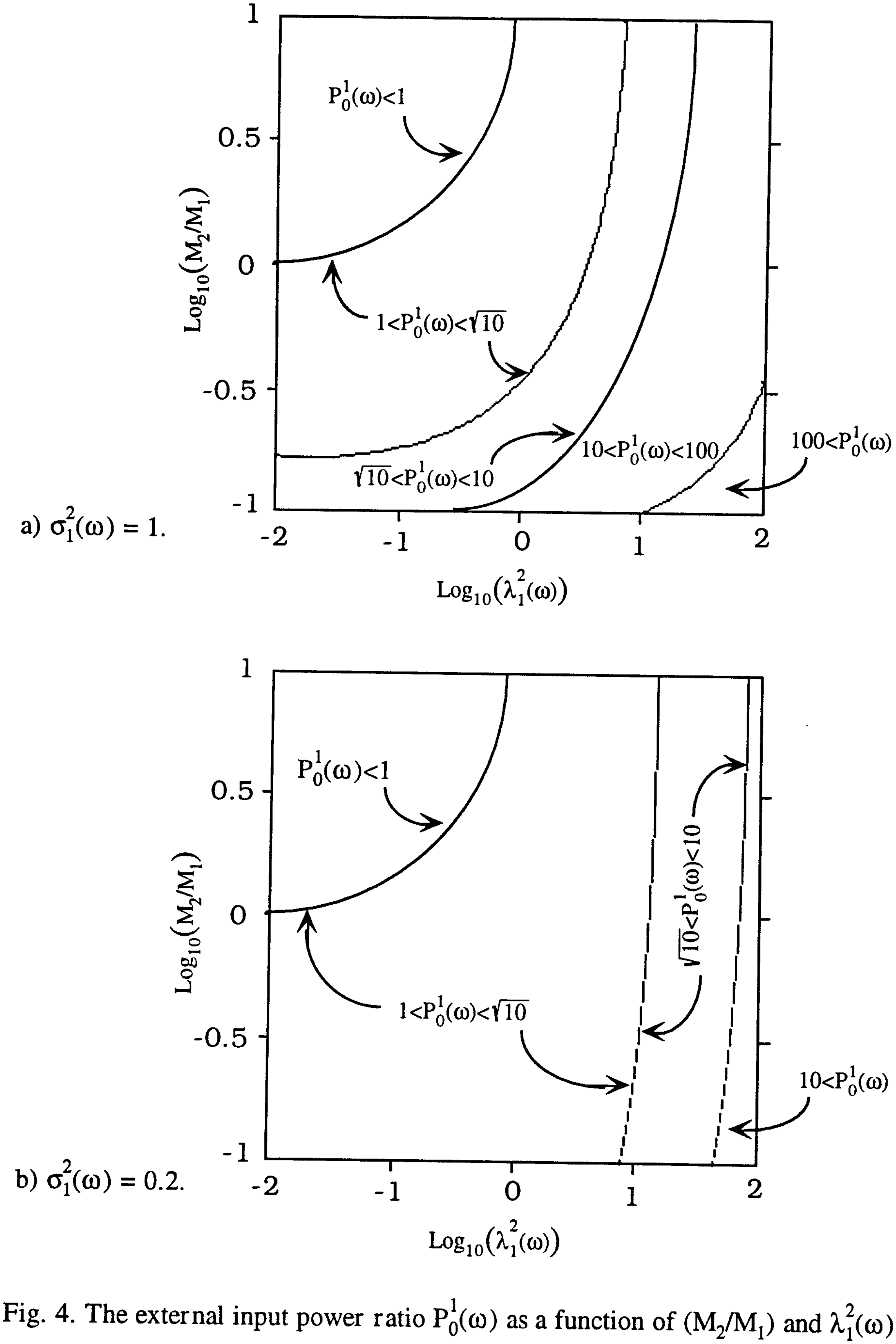


Fig. 4. The external input power ratio $P_0^1(\omega)$ as a function of (M_2/M_1) and $\lambda_1^2(\omega)$ for a fixed value of the modal energy ratio, $\sigma_1^2(\omega)$.

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